Equilibrium in a Model of Production Networks

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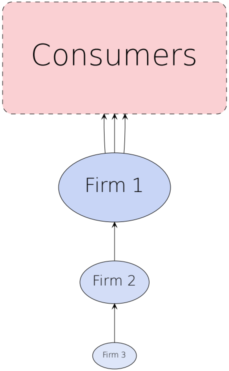
Outline

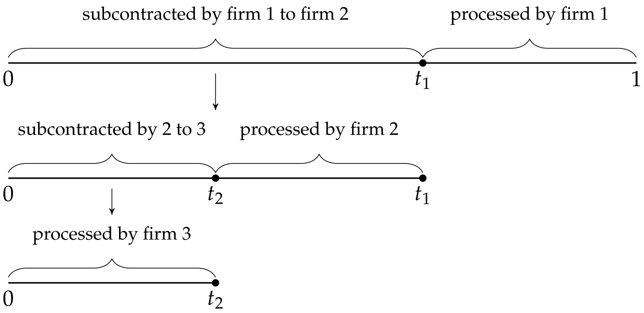
Production Chains

- Kikuchi, Nishimura, and Stachurski (2018) ("Span of control, transaction costs, and the structure of production chains")
- Assumptions:
    1. Cost function c increasing and strictly convex
    2. Transaction cost $\delta > 1$
       Firm 2 sells at p; Firm 1 pays δp



Production Chains

- Source: Kikuchi, Nishimura, and Stachurski (2018)



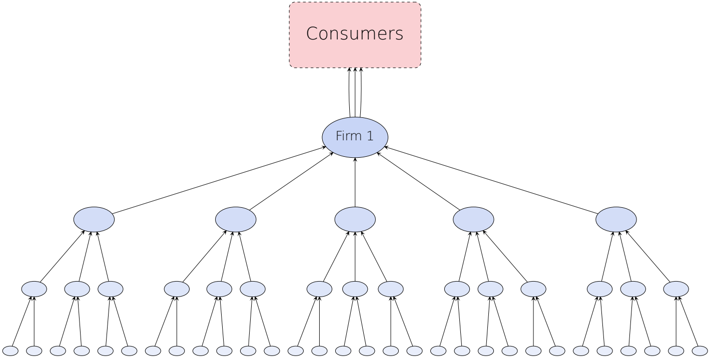
- The firm at stage s chooses t to minimize cost:

$$\min_{t \leq s} \{c(s-t) + \delta p(t)\}$$

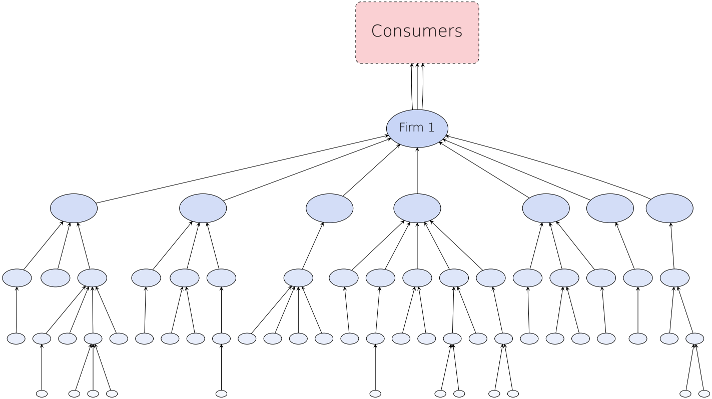
- For example, firm 2's total cost: $c(t_1 - t_2) + \delta p(t_2)$
- Equilibrium price:

$$p^*(s) = \min_{t \leq s} \{c(s-t) + \delta p^*(t)\}$$

Production Networks



Production Networks



Main Results

- Uniqueness of the equilibrium price function
- An algorithm for faster computation
- Extension to the stochastic case

Model

- The stage of production is index by $s \in [0,1]$
- Each firm faces
  1. price function $p : [0,1] \to \mathbb{R}_+$
  2. cost function $c : [0,1] \to \mathbb{R}_+$
  3. transaction costs $\delta > 1$ and $g : \mathbb{N} \to \mathbb{R}_+$
- Assumptions:
  1. c differentiable, increasing, and strictly convex
  2. $c(0) = 0$ and $c'(0) > 0$
  3. $g(1) = 0$, g strictly increasing, and $g(k) \to \infty$ as $k \to \infty$
- Firm at stage s minimizes cost

$$\min_{\substack{t \leq s \\ k \in \mathbb{N}}} \{c(s-t) + g(k) + \delta k p(t/k)\}$$

Equilibrium

- Equilibrium price p^* satisfies

$$p^*(s) = Tp^*(s) := \min_{\substack{t \leq s \\ k \in \mathbb{N}}} \{c(s-t) + g(k) + \delta k p^*(t/k)\}$$

- Find the fixed point of operator T
- Problem: contraction mapping doesn't work because $\delta > 1$

Monotone Concave Operator Theory

Basic Idea

- A theorem due to Du (1989)
- Consider a function T defined on $[u_0, v_0]$. If
    1. T is increasing and concave
    2. $T(u_0) > u_0$
    3. $T(v_0) < v_0$
- Then
    1. T has a unique fixed point x^* in $[u_0, v_0]$
    2. $T^n(x) \to x^*$ for all $x \in [u_0, v_0]$

Monotone Concave Operator Theory

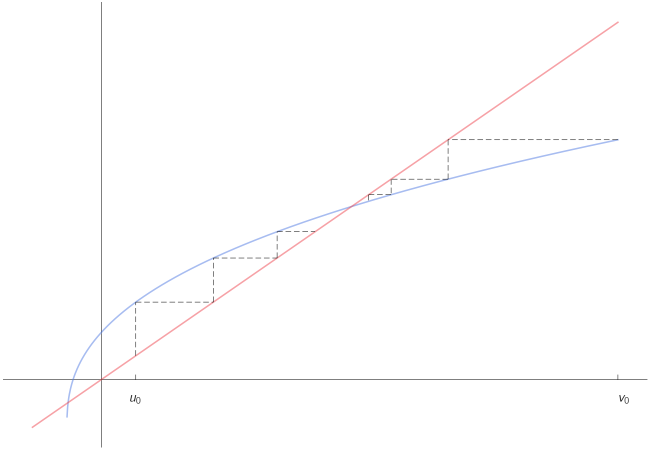


Figure 1: A Simple Example in $\mathbb{R}$

Monotone Concave Operator Theory

General Version (Du 1989)

- Theorem 3.1 in the paper
- T is defined on any partially ordered Banach space
  1. $[u_0, v_0]$: an order interval
  2. Similar definitions of increasing and concave operators
  3. Other technical conditions on the Banach space
- $T^n(x) \to x^*$ in norm

Uniqueness Result

Theorem 3.2

Let $u_0(s) = c'(0)s$, $v_0(s) = c(s)$, and $[u_0, v_0]$ be the order interval on $C([0,1])$ with the usual partial order. If the above assumptions hold, then T has a unique fixed point p^* in $[u_0, v_0]$. Furthermore, $T^n p \to p^*$ for any $p \in [u_0, v_0]$.

$$p^*(s) = Tp^*(s) = \min_{\substack{t \leq s \\ k \in \mathbb{N}}} \{c(s-t) + g(k) + \delta k p^*(t/k)\}$$

Equilibrium Price

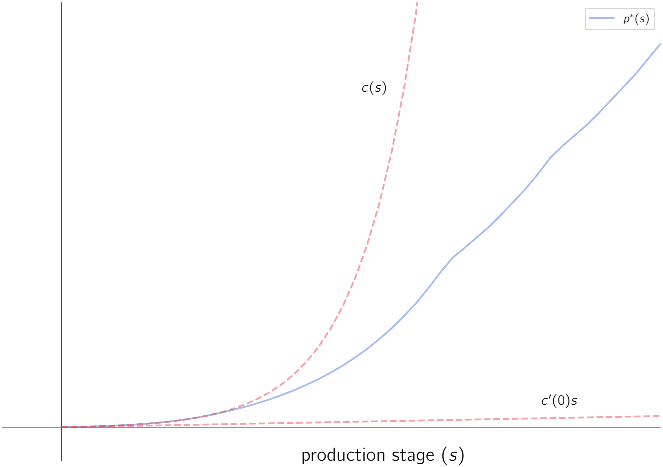


Figure 2: An example of equilibrium price function

Computation

- One way to compute p^* is pick any $p \in [u_0, v_0]$ and compute $T^n p$ with large n: usually slow
- A faster algorithm
  1. Choose grid points $\{0, h, 2h, \dots, 1\}$
  2. Set $p(0) = 0$ and $s = h$
  3. Repeat:
     i Set $p(s)$ by
     $$p(s) = \min_{\substack{t \leq s-h \\ k \in \mathbb{N}}} \{c(s-t) + g(k) + \delta k p(t/k)\}$$
     ii Define p on $[0, s]$ by linear interpolation
     iii Set $s = s + h$; if $s > 1$, stop
- Advantage: for a certain number of grid points, the number of optimizations required is fixed

Convergence

Theorem 4.1

As the number of grid points goes to infinity, the price functions computed from the algorithm converge to p^* uniformly.

Speed

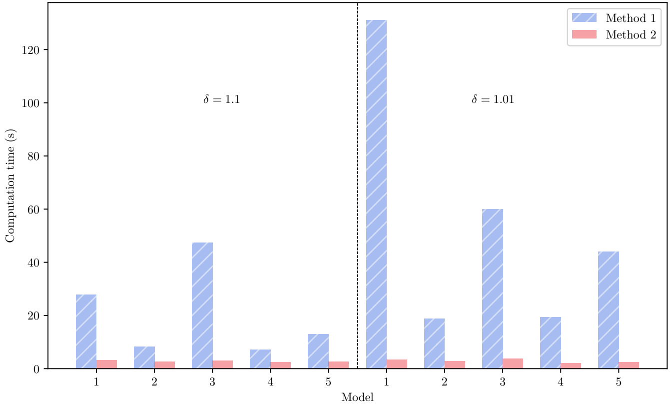


Figure 3: Speed comparison

Stochastic Case

- What if firms cannot choose the exact number of upstream partners?
- We assume each firm chooses a "search effort" λ: the resulting number of partners follows a Poisson distribution

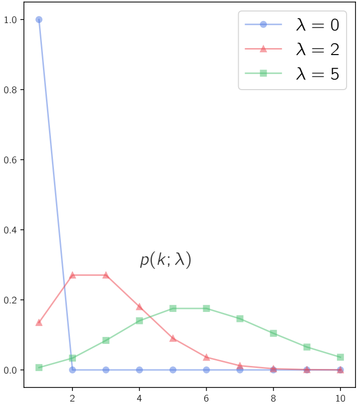


Figure 4: Distributions for different λ

Equilibrium

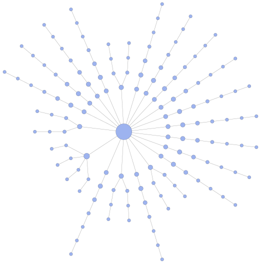
- New equilibrium price:

$$p^*(s) = \min_{\substack{t \leq s \\ \lambda \geq 0}} \left\{ c(s-t) + \mathbb{E}_k^{\lambda} \left[g(k) + \delta k p^*(t/k) \right] \right\}$$

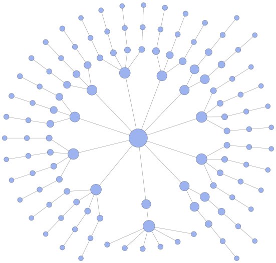
- Under the same assumptions, uniqueness holds and the algorithm works

Production Networks with Uncertainty

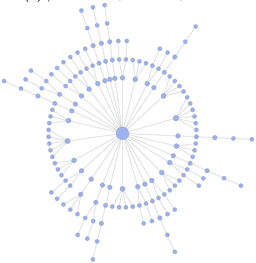
Specification: $c(s) = s^{\theta}$, $g(k) = \beta(k-1)^{1.5}$



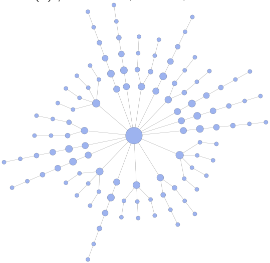
(a) $\beta = 0.0005$, $\delta = 1.05$, $\theta = 1.2$



(b) $\beta = 0.0005$, $\delta = 1.1$, $\theta = 1.2$



(c) $\beta = 0.0001$, $\delta = 1.05$, $\theta = 1.2$



(d) $\beta = 0.0005$, $\delta = 1.05$, $\theta = 1.15$

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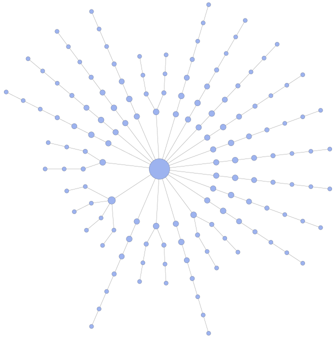

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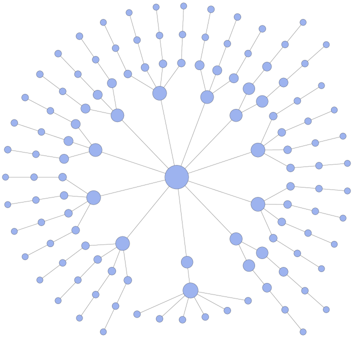

(b) $\delta = 1.1$

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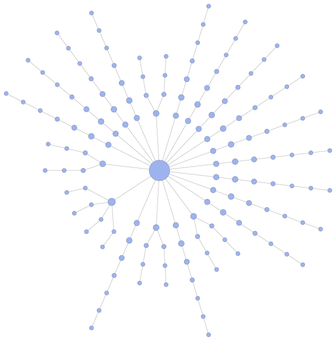

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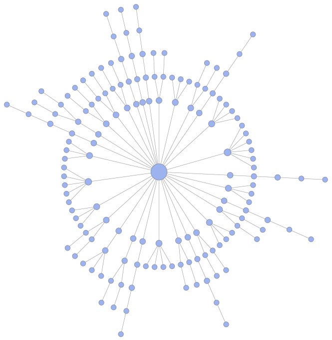

(c) $\beta = 0.0001$

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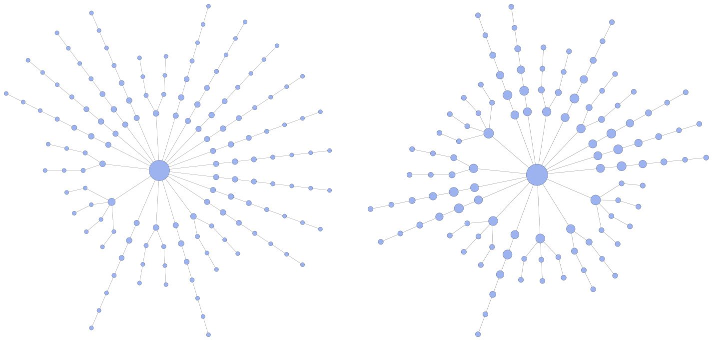

(a) $\theta = 1.2$ (d) $\theta = 1.15$

References

Kikuchi, Tomoo, Kazuo Nishimura, and John Stachurski. 2018.
Span of control, transaction costs, and the structure of production chains
Theoretical Economics 13(2), 729–760.

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